Tribhuvan University

2072

Bachelor Level 4 Yrs. Prog./Ist Year/Science & Tech. Full Marks: 75

Analytical Geometry & Vector Analysis (Math. 102) Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt ALL the questions.

Group "A" 5×7=35

1. Explain the auxiliary circle and the eccentric angle of a point in an ellipse. Find the point at which the line $\ell x + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also, find the condition for the line to be a normal to the ellipse. [3+2+2]

2. Define the tangent and normal to a conic. Find the equation of tangent at my point (x_1, y_1) of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [2+5]

   OR

   Write the condition for the second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent different types of conics. If the centre of the hyperbola $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is at (h, k) then prove that the pair of asymptotes are given by $f(x, y) = f(h, k)$. [3.5+3.5]

3. Obtain the expression for the angle between a line with direction ratios ℓ, m, n and a plane $ax + by + cz + d = 0$. Find the points at which the line $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ cuts surface $11x^2 - 5y^2 + z^2 = 0$. [4+3]

4. Write the equation of a sphere in diameter form. Find the equation of a sphere that cuts each positive coordinate axes at a unit distance and the radius as small as possible. [1+6]

   OR

   What is a plane section of a sphere? Find the centre and the radius of the circle $x^2 + y^2 + z^2 + 12x - 12y - 16z + 111 = 0 = 2x + 2y + z - 17$. [2+5]

5. Prove that in a scalar triple product, the position of the dot and the cross can be interchanged. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$ if $\vec{b}$ and $\vec{c}$ are not parallel. [2.5+2.5+2.5]

GROUP "B" 10×4=40

6. Find the transformed equation of the curve $9x^2 + 4y^2 + 18x - 16y = 11$, if the origin is shifted at (-1, 2) but the direction of axes are not changed. [4]

7. Find the condition for the line $x\cos\alpha + y\sin\alpha = p$ will be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. [4]

8. If PSP′ and QSQ′ are two perpendicular focal chords of a conic $\frac{l}{r} = 1 + e\cos\theta$, prove that $\frac{1}{PP'} + \frac{1}{QQ'}$ is constant, where S is the focus.

OR [4]

Find the equation of tangent at point whose vectional angle is α for the conic $\frac{l}{r} = 1 + e\cos\theta$.

9. Find the equation of the plane passing through the point (1, -2, 3) and perpendicular to the line passing through (3, 4 - 5) and (1, 2, 3). [4]

10. Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $x - 4 = \frac{y+3}{-4} = \frac{Z+1}{7}$. [4]

11. If the section of a cone with vertex at L and guiding curve, the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; z = 0$ by the xy - plane is a rectangular hyperbola, find the locus of L.

OR [4]

Find the equation of the right cylinder that passes through $y^2 = 4ax$, $z = 0$ and whose generators are parallel to the line $x = y = z$.

12. Find the equation of the tangent plane at the point (*f*, g, h) to the conicoid $ax^2 + by^2 + cz^2 = 1$.

OR [4]

P.T.O.

If $3x + 12y - 6z = 17$ is a tangent plane to the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$, find the point of contact.

13. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then show that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are also non-coplanar. [4]

14. For the space curve $x = 3t$, $y = 3t^2$, $z = 2t^3$, prove that

$$\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right] = 216.$$ [4]

15. Find the unit vector normal to the surface $z = x^2 + y^2$ at the point (-1, -2, 5).

OR [4]

For any space vector $\vec{v}$ prove that

grad (div $\vec{v}$) = curl (curl $\vec{v}$) + $\sum \frac{\partial^2 \vec{v}}{\partial x^2}$. [4]