POKHARA UNIVERSITY

Level: Bachelor	Semester – Spring	Year : 2010
Programme: BE		Full Marks: 100
Course: Engineering Mathematics IV		Pass Marks: 45
		Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define analyticity of the complex valued function f(z). Show that the necessary condition for analyticity of the complex valued function f(z) =u + iv is $u_x = v_y$ and $u_y = -v_x$ at any point (x , y). 7

   b) Evaluate the following integrals using Cauchy's integral formula: 8

   i) $\oint_C \frac{z+1}{(z^3-4z)}dz$, C: |z -2| = 3/2

   ii) $\oint_C \frac{z+1}{(z^3-2z^2)}dz$, C is the unit circle.

2. a) Define zeros and poles of the complex valued function f(z). Let z_0 is a pole of the function f(z) with order n, derive the residue of f(z) at z $= z_0$. Evaluate $\oint_C \frac{\sinh z}{(2z-i)}dz$, where C: the circle $|z-i| = 1$, in counter clockwise direction. 8

OR

State Laurent's series. Expand the function f(z) = $\frac{1}{z-z^3}$ in the region 1</z-1/<2.

b) Find the solution of one dimensional wave equation corresponding to the triangular initial deflection: 7

$$f(x) = \begin{cases} \frac{2K}{L} x, \text{ if } 0 < x < L/2 \\ \frac{2K}{L} (L - x), \text{ if } L/2 < x < L \end{cases}$$

with its initial velocity zero and c = 1

3. a) Find u(x ,t) from one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, with boundary condition $u(0,t) = 0 = u(L,t)$, initial deflection $f(x)$ and initial velocity $\left.\frac{\partial u}{\partial t}\right|_{t=0}$ = g(x). 8

b) Find the Laplacian in cylinder polar coordinates. 7

OR

Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary assumptions. 7

4. a) State and prove the first shifting theorem of Z-transform. Hence find 8

$Z(e^{-at}.t)$ *and* $Z(e^{-at})$.

b) If $Z[f(n)] = F(z)$, then show that $Z[f(n+k)] = z^k[F(z) - \sum_{n=0}^{k-1} f(n)z^{-n}]$. Then find the expression of $Z[f(n+1)]$ and $Z[f(n+2)]$. 7

5. a) Using Z-transform solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n \text{ when } y_0 = y_1 = 0.$ 7

b) Maximize $z = 4x_1 + x_2 + 2x_3$, subject to $x_1 + x_2 + x_3 \leq 1$, $x_1 + x_2 - x_3 \leq 0$, $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$ by using simplex methods. 8

6. a) Derive Fourier integral in complex form. Also derive the formula for Fourier transform and their inversion. 8

OR

Verify the convolution theorem for the functions $f(x) = e^{-x^2}$ and $g(x) = e^{-x^2}$.

b) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}, a > 0.$ 7

OR

Find Fourier cosine integral representation of the function $f(x) = e^{-kx}$ for k > 0. Using it, find Fourier cosine transform of the function $f(x) = \frac{1}{1+x^2}$.

7. Attempt the following questions**:**

a) Represent the equation $x^2 + y^2 = 9, z = 5\tan^{-1}(y/x)$ parametrically. 2

b) Find unit tangent vector on the curve C, whose position vector is $\vec{r} = \cos t\,\vec{i} + 2\sin t\,\vec{j}$, at the point $P\left(\frac{1}{2}, \sqrt{3}, 0\right)$. 2

c) Sketch the feasible region of the inequalities 2
$-0.5x_1 + x_2 \le 2, \quad x_1 + x_2 \ge 2,$
$-x_1 + 5x_2 \ge 5, \quad x_1 \ge 0, \quad x_2 \ge 0.$

d) Show that $F_c\{af(t) + bg(t)\} = aF_c\{f(t)\} + bF_c\{g(t)\}$ where F_c is the Fourier cosine transform. 2

e) Find the solution of the partial differential equation $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$, by separation of variables methods. 2